Partial Differentiation

The functions studied so far are of a single independent variable. There are functions which depends on two or more variables. Example, the pressure(P) of a given mass of gas is dependent on its volume(v) and temperature (T).

Functions of two variable

A function f : X x Y to Z is a function of two variables if there exist a unique element z = f(x,y) in Z corresponding to every pair (x,y) in X x Y.

Domain of f is $X \times Y$.

$f(X \times Y)$ is the range of f. { $f(X \times Y)$ C Z }

Notation : - z = f(x,y) means z is a function of two variables x and y.

Limit of a function of two variable

A function f(x,y) tends to limit l as (x,y) → (a,b), .If given $\epsilon > 0$, there exist δ> 0 such that |*f(x,y)-l* | <ϵ whenever o < |(x,y) – (a,b)| <δ .

Continuity

A function f(x,y) is said to be continuous at a point (a,b) if

(i) f(a,b) is defined

(ii) $\lim_{(x,y)\to(a,b)} f(x,y)$ exists.

(iii) $\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$

Finding limits and testing continuity of functions of two variable is beyond our syllabus so we have to skip these topics here.

Partial derivatives

Let z = f(x,y) be function of two variables.

If variable x undergoes a chance δx, while y remains constant, then z undergoes a changes written as δz

Now, δz = f(x+ δx,y) – f(x,y)

If $\frac{\delta z}{\delta x}$ exist as δx→ 0, then we write the partial derivative of z w.r.t x as

$$\frac{\partial f}{\partial x} = \frac{\partial z}{\partial x} = f_x = z_x = \lim_{\delta x \to 0} \frac{f(x+\delta x,y)-f(x,y)}{\delta x}$$

Similarly partial derivative of z w.r.t y ,

$$\frac{\partial f}{\partial y} = \frac{\partial z}{\partial y} = f_y = z_y = \lim_{\delta y \to 0} \frac{f(x,y+\partial y)-f(x,y)}{\delta y}$$

$\frac{\partial}{\partial x}$, $\frac{\partial}{\partial y}$ symbols are used to notify the partial differentiation.

Note

As from above theory it is clear when partial differentiation w.r.t x is taken , then y is treated as constant and vice – versa. (All the formulae and techniques used in derivative chapter remain same here)

2nd Order Partial Differentiation

If we differentiate the $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ w.r.t x or y , then we set higher order partial derivatives as follows.

1st Order Partial Derivatives $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$.

2nd Order Partial Derivatives

$\frac{\partial^2 Z}{\partial x^2} = \frac{\partial}{\partial x} (\frac{\partial z}{\partial x}) = z_{xx} = f_{xx}$

$\frac{\partial^2 Z}{\partial y \partial x} = \frac{\partial}{\partial y} (\frac{\partial z}{\partial x}) = z_{yx} = f_{yx}$

$\frac{\partial^2 Z}{\partial x \partial y} = \frac{\partial}{\partial x} (\frac{\partial z}{\partial y}) = z_{xy} = f_{xy}$

$\frac{\partial^2 Z}{\partial y^2} = \frac{\partial}{\partial y} (\frac{\partial z}{\partial y}) = z_{yy} = f_{yy}$

Note: $f_{yx} = f_{xy}$ *when partial derivatives are continuous .*

Example -1

Find $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$

(i) z = $2x^2y + xy^2 + 5xy$.

(ii) z = $\tan^{-1}(\frac{x}{y})$ **[2018-S]**

(iii) z = $e^y \tan x$ **[2019-W]**

(iv) z = log $(x^2 + y^2)$ **[2015-S]**

(v) z = $\sin^{-1}(\frac{x}{y})$ **[2014-S]**

(vi) z = f$(\frac{y}{x})$ **[2017-S]**

(vii) $x^y + y^x$

Ans.

(i) z = $2x^2y$ + xy^2 + 5xy

$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}(2x^2y) + \frac{\partial}{\partial x}(xy^2) + \frac{\partial}{\partial x}(5xy)$ (Here y is treated as constant)

$= 2y\frac{\partial}{\partial x}(x^2) + y^2\frac{\partial}{\partial x}(x) + 5y\frac{\partial}{\partial x}(x)$

$= 2y.2x + y^2.1 + 5y.1$

$= 4xy + y^2 + 5y.$

$\frac{\partial z}{\partial y} = 2x^2\frac{\partial y}{\partial y} + x\frac{\partial y^2}{\partial y} + 5x\frac{\partial y}{\partial y}$

$= 2x^2 + x.2y + 5x = 2x^2 + 2xy + 5x$

(ii) z = $\tan^{-1}(\frac{x}{y})$

$\frac{\delta z}{\delta x} = \frac{1}{1+(\frac{x}{y})^2}\frac{\partial}{\partial x}(\frac{x}{y}) = \frac{1}{\frac{y^2+x^2}{y^2}} \cdot \frac{1}{y}$

$= \frac{y^2}{y(x^2+y^2)} = \frac{y}{x^2+y^2}$

$\frac{\partial z}{\partial y} = \frac{1}{1+(\frac{x}{y})^2}\frac{\partial}{\partial y}(\frac{x}{y}) = \frac{1}{\frac{y^2+x^2}{y^2}}(\frac{-x}{y^2})$

$= \frac{-x}{x^2+y^2}$

(iii) z = $e^y \tan x$

$\frac{\partial z}{\partial x} = e^y\frac{\partial}{\partial x}(\tan x) = \frac{e^y}{1+x^2}$

$\frac{\partial z}{\partial y} = \frac{\partial(e^y)}{\partial x}\tan x = e^y \tan x$

(iv) z = log $(x^2 + y^2)$

$\frac{\partial z}{\partial x} = \frac{1}{(x^2+y^2)}\frac{\partial}{\partial x}(x^2 + y^2) = \frac{2x}{x^2+y^2}$ $\{\frac{\partial}{\partial x}y^2 = 0$ As y is constant$\}$

$\frac{\partial z}{\partial y} = \frac{1}{(x^2+y^2)}\frac{\partial}{\partial y}(x^2 + y^2) = \frac{2y}{x^2+y^2}$

(v) z = $\sin^{-1}(\frac{x}{y})$

$\frac{\delta z}{\delta x} = \frac{1}{\sqrt{1-\frac{x^2}{y^2}}}\frac{\partial}{\partial x}(\frac{x}{y}) = \frac{1}{\sqrt{\frac{y^2-x^2}{y^2}}}.\frac{1}{y} = \frac{1}{\sqrt{y^2-x^2}}$

$\frac{\partial z}{\partial y} = \frac{1}{\sqrt{1-\frac{x^2}{y^2}}}\frac{\partial}{\partial y}(\frac{x}{y}) = \frac{1}{\sqrt{\frac{y^2-x^2}{y^2}}}.(\frac{-x}{y^2})$

$= -\frac{x}{y\sqrt{y^2-x^2}}$

(vi) z = f$(\frac{y}{x})$

$\frac{\partial z}{\partial x}$ = f ‘ $(\frac{y}{x})\frac{\partial}{\partial x}(\frac{y}{x})$ = f ‘ $(\frac{y}{x})$. $(\frac{-y}{x^2})$

$= \frac{-y}{x^2}$ f ‘ $(\frac{y}{x})$

$\frac{\partial z}{\partial y}$ = f ‘ $(\frac{y}{x})\frac{\partial}{\partial y}(\frac{y}{x})$ = f ‘ $(\frac{y}{x}).\frac{1}{x}$

$= \frac{1}{x}$ f ‘ $(\frac{y}{x})$

vii) z = $x^y + y^x$

$\frac{\partial z}{\partial x}$ = y x^{y-1} + $y^x \ln y$ (y is a constant here)

$\frac{\partial z}{\partial y}$ = x^yln x + x y^{x-1} (As x is treated as constant)

Example-2 . Find f_{xx} and f_{yx} where f(x,y) = $x^3 + y^3 + 3xy$

Ans: - $f_x = 3x^2 + 3y$, $f_y = 3y^2 + 3x$

$f_{xx} = \frac{\partial}{\partial x}(f_x) = 6x + 0 = 6x$

$f_{yx} = \frac{\partial}{\partial y}(f_x) = 0 + 3 = 3$

Example – 3

If z = log $(x^2 + y^2)$ + $\tan^{-1}(\frac{y}{x})$, prove that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$

Ans. $\frac{\partial z}{\partial x} = \frac{1}{(x^2+y^2)} 2x + \frac{1}{1+\frac{y^2}{x^2}}(-\frac{y}{x^2})$

$= \frac{2x}{x^2+y^2} + \frac{x^2}{x^2+y^2}(-\frac{y}{x^2}) = \frac{2x-y}{x^2+y^2}$

$\frac{\partial z}{\partial y} = \frac{1}{(x^2+y^2)} 2y + \frac{1}{1+\frac{y^2}{x^2}} (\frac{1}{x})$

$= \frac{2y}{(x^2+y^2)} + \frac{x^2}{(x^2+y^2).x}$

$= \frac{2y+x}{(x^2+y^2)}$

Now $\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}(\frac{\partial z}{\partial x}) = \frac{(x^2+y^2)(2-0)-(2x-y)(2x+0)}{(x^2+y^2)^2}$

$= \frac{2x^2+2y^2-4x^2+2xy}{(x^2+y^2)^2} = \frac{2y^2-2x^2+2xy}{(x^2+y^2)^2}$

$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}(\frac{\partial z}{\partial y}) = \frac{(x^2+y^2)(2+0)-(2y+x)(0+2y)}{(x^2+y^2)^2}$

$= \frac{2x^2+2y^2-4y^2-2xy}{(x^2+y^2)^2} = \frac{2x^2-2y^2-2xy}{(x^2+y^2)^2}$

Now $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{2y^2-2x^2+2xy+2x^2-2y^2-2xy}{(x^2+y^2)^2}$

$= \frac{0}{(x^2+y^2)^2} = 0$ (Proved)

Homogenous function and Euler's theorem

Homogenous function

A function f (x,y) is said to be homogenous in x and y of degree n iff (tx, ty) = $t^n f(x,y)$ where t is any constant.

Example – 4

Test whether the following functions are homogenous or not. If homogenous then find their degree.

(i) 2xy^2 + 3x^2y (ii) $\sin^{-1}(\frac{x}{y})$

(iii) $\frac{3x^2+2y^2}{x+y}$ (iv) $x^2 + 2xy + 4x$

Ans.

(i) Let f(x, y) = $2xy^2$ + $3x^2y$

f(tx, ty) = $2(tx)(ty)^2 + 3\,(tx)^2(ty)$

$= 2txt^2y^2 + 3\,t^2x^2ty$

$= t^3(2xy^2 + 3x^2y) = t^3 f(x,y)$

Hence f(x,y) is a homogenous function of degree 3.

(ii) Let f(x, y) = $\sin^{-1}(\frac{x}{y})$

f(tx, ty) = $\sin^{-1}(\frac{tx}{ty}) = \sin^{-1}\frac{x}{y} = t^0 \sin^{-1}(\frac{x}{y}) = t^0 f(x, y)$

Hence f(x, y) is a homogenous function of degree '0'.

(iii) f(x, y) = $\frac{3x^2+2y^2}{x+y}$

f(tx, ty) = $\frac{3(tx)^2+2(ty)^2}{tx+ty} = \frac{t^2}{t}\frac{(3x^2+2y^2)}{x+y}$ = t f(x, y)

Hence f (x, y) is a homogenous function of degree 1.

(iv) f(x , y) = $x^2 + 2xy + 4x$

f(tx, ty) = $(tx)^2$ + 2 (tx) (ty) + 4(tx)

= t (tx^2 + 2t xy + 4x)

So here f (tx, ty) cannot be expressed as $t^n f(x,y)$

Hence f(x,y) is not a homogenous function.

Note

(i) *If each term in the expression of a function is of the same degree then the function is homogenous.*

(ii) *If z is a homogenous function of x and y of degree n , then $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ are also homogenous of degree n-1.*

(iii) *If z = f(x, y) is a homogenous function of degree n , then we can write it as*

z = $x^n \Phi(\frac{y}{x})$

e.g. In example - 4(i) $2xy^2 + 3x^2y$ is homogenous function of degree 3.

Now $f(x,y) = 2xy^2 + 3x^2y = x^3(2(\frac{y}{x})^2 + 3\,(\frac{y}{x})) = x^3\Phi(\frac{y}{x})$

Similarly in Example – 4 (iii), f(x,y) is of degree 1.

Now $f(x, y) = \frac{3x^2+2y^2}{x+y} = \frac{x^2}{x}(\frac{3+2(\frac{y}{x})^2}{1+\frac{y}{x}}) = x(\frac{3+2(\frac{y}{x})^2}{1+\frac{y}{x}}) = x\,\Phi(\frac{y}{x})$

Euler's theorem

If z is a homogenous function of degree n , then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = n z$. **[2014-S]**

Proof: -

Since z is a homogenous function of degree n , so z can be written as

$$z = x^n \Phi(\frac{y}{x})$$

Now $\frac{\partial z}{\partial x}$ = n x^{n-1} $\Phi(\frac{y}{x})$ + $x^n \Phi'(\frac{y}{x}) . \frac{\partial}{\partial x}(\frac{y}{x})$

= n x^{n-1} $\Phi(\frac{y}{x})$ + $x^n \Phi'(\frac{y}{x})$ (- $\frac{y}{x^2}$)

= n x^{n-1} $\Phi(\frac{y}{x})$ - x^{n-2} y $\Phi'(\frac{y}{x})$ ------------ (1)

Similarly, $\frac{\partial z}{\partial y} = x^n \Phi'(\frac{y}{x}) \frac{\partial}{\partial y}(\frac{y}{x})$

= $x^n \Phi'(\frac{y}{x})$ $(\frac{1}{x})$ = x^{n-1} $\Phi(\frac{y}{x})$ ----------- (2)

Now x × Equation (1) + y × Equation (2)

$\Rightarrow x \frac{\partial z}{\partial x}$ + y $\frac{\partial z}{\partial y}$ = x {n x^{n-1} $\Phi(\frac{y}{x})$ - x^{n-2} y $\Phi'(\frac{y}{x})$ } + y x^{n-1} $\Phi'(\frac{y}{x})$

= n $x^n \Phi(\frac{y}{x})$ - $x^{n-1} y$ $\Phi'(\frac{y}{x})$ + x^{n-1} y $\Phi'(\frac{y}{x})$

= n $x^n \Phi(\frac{y}{x})$ = nz (proved)

Example – 5

Verify Euler's theorem for z = $\frac{y}{x}$ **[2014-S]**

Ans. $\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}(\frac{y}{x}) = -\frac{y}{x^2}$

$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}(\frac{y}{x}) = \frac{1}{x}$

Here z = f(x , y) = $\frac{y}{x}$

F(tx, ty) = $\frac{ty}{tx} = \frac{y}{x}$ = f(x, y)

Hence f(x, y) is a homogenous function of degree 0.

Statement of Euler's theorem is $x \frac{\partial z}{\partial x}$ + y $\frac{\partial z}{\partial y}$ = n z (here n=0)

=>$x \frac{\partial z}{\partial x}$ + y $\frac{\partial z}{\partial y}$ = 0 . Z = 0

Now we have to verify it.

From above

L.H.S = $x\frac{\partial z}{\partial x}$ + y $\frac{\partial z}{\partial y}$ = x (- $\frac{y}{x^2}$) + y . $\frac{1}{x}$ = - $\frac{y}{x}$ + $\frac{y}{x}$ = 0 = R.H.S

Hence Euler's theorem is verified.

Example – 6

Verify Euler's theorem for z = x^2y^2 + 4xy^3 - 3x^3y

Ans. Here z = f(x, y) = x^2y^2 + 4xy^3 - 3x^3y

F (tx, ty) = $t^2x^2t^2y^2$ + 4txt^3y^3 - 3t^3x^3y

= t^4(x^2y^2 + 4xy^3 - 3x^3y) = t^4f(x, y)

Hence z is homogenous function of degree 4.

Here n = 4.So, the statement of Euler's theorem is

$x\frac{\partial z}{\partial x}$ + y $\frac{\partial z}{\partial y}$ = 4z

Now we have to verify it

$\frac{\partial z}{\partial x}$ = $\frac{\partial}{\partial x}$($x^2y^2$ + 4xy^3 - 3x^3y)

= $2xy^2$ + 4y^3 - 3 (3x^2)y

= $2xy^2$ + 4y^3 - 9x^2y ----------------- (1)

$\frac{\partial z}{\partial y}$ = $\frac{\partial}{\partial y}$($x^2y^2$ + 4xy^3 - 3x^3y)

= 2x^2y + 12xy^2 - 3x^3 ------------------ (2)

L.H.S = x $\frac{\partial z}{\partial x}$ + y $\frac{\partial z}{\partial y}$

= x (2x y^2 + 4y^3 - 9x^2y) + y (2x^2y + 12xy^2 - 3x^3) {from (1) and (2)}

= 2x^2y^2 + 4xy^3 - 9 x^3y + 2x^2y^2 + 12xy^3 - 3x^3y

= 4x^2y^2 + 16xy^3 - 12x^3y

= 4 (x^2y^2 + 4xy^3 - 3x^3y) = 4z (verified)

Example – 7

If z = $\sin^{-1}(\frac{xy}{x+y})$ show that x $\frac{\partial z}{\partial x}$ + y $\frac{\partial z}{\partial y}$ = tan z **[2017-S, 2018-S, 2019-W]**

Ans.

Let z = $\sin^{-1}(\frac{xy}{x+y})$ = $\sin^{-1}u$

Now u = $\frac{xy}{x+y}$

u = (tx , ty) = $\frac{txty}{tx+ty}$ = $\frac{t^2}{t}(\frac{xy}{x+y})$ = tu

Hence u is homogenous function of degree 1.

So by Euler's theorem

x $\frac{\partial u}{\partial x}$ + y $\frac{\partial u}{\partial y}$ = 1 . u = u ----------- (1)

As z = $\sin^{-1} u$

⇒ u = sin z

$\frac{\partial u}{\partial x}$ = $\frac{\partial}{\partial x}$(sin z) = cos z $\frac{\partial z}{\partial x}$ ------------ (2)

And $\frac{\partial u}{\partial y}$ = $\frac{\partial}{\partial y}$(sin z) = cos z $\frac{\partial z}{\partial y}$ ---------- (3)

From (1), (2) and (3)

⇒ x cos z $\frac{\partial z}{\partial x}$ + y cos z $\frac{\partial z}{\partial y}$ = sin z

⇒$x \frac{\partial z}{\partial x}$ + y $\frac{\partial z}{\partial y}$ = $\frac{\sin z}{\cos z}$ = tan z (Proved)

Example – 8

If u = $\sin^{-1}(\frac{x}{y})$ + $\tan^{-1}(\frac{y}{x})$ show that x $\frac{\partial u}{\partial x}$ + y $\frac{\partial u}{\partial y}$ = 0

Ans.

If u = $\sin^{-1}(\frac{x}{y})$ + $\tan^{-1}(\frac{y}{x})$

u (tx, ty) = $\sin^{-1}(\frac{tx}{ty})$ + $\tan^{-1}(\frac{ty}{tx})$

= $\sin^{-1}(\frac{x}{y})$ + $\tan^{-1}(\frac{y}{x})$

= u (x , y)

Hence u is a homogenous function of degree '0'

So by Euler's theorem

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$$

Example – 9

If $z = \tan^{-1}(\frac{x^3+y^3}{x+y})$, show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \sin 2z$ **[2017-w]**

Ans. Let $z = \tan^{-1}$, where $u = (\frac{x^3+y^3}{x+y})$

Now $u(tx, ty) = \frac{t^3x^3+t^3y^3}{tx+ty} = t^2 (\frac{x^3+y^3}{x+y}) = t^2u$

Hence u is a homogenous function of degree 2.

So by Euler's theorem

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u$ ----------- (1)

Now $z = \tan^{-1} u$

$\Rightarrow u = \tan z$ -------------------------------- (2)

$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(\tan z) = sec^2z\frac{\partial z}{\partial x}$ -------------- (3)

And $\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}(\tan z) = sec^2z\frac{\partial z}{\partial y}$ ---------(4)

From (1), (2), (3) and (4)

$\Rightarrow x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u$

$\Rightarrow x sec^2z\frac{\partial z}{\partial x} + y\, sec^2z\frac{\partial z}{\partial y} = 2\tan z$

$\Rightarrow x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2\frac{\tan z}{sec^2z} = 2\tan z\, cos^2z$

$= 2\frac{\sin z}{\cos z}cos^2z = 2\sin z\cos z$

$= \sin 2z$ (proved)

Example – 10

If z is a homogenous function of x and y of degree n and $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ are continuous , then show that

$x^2 \frac{\partial^2 z}{\partial x^2}$ + 2xy $\frac{\partial^2 z}{\partial x \partial y}$ + $y^2 \frac{\partial^2 z}{\partial y^2}$ = n(n-1)z

Proof

Given z is a homogenous function of degree n

So by Euler's theorem

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = nz \text{ ------------ (1)}$$

Differentiating (1) w.r.t x ,

$1. \frac{\partial z}{\partial x} + x \frac{\partial^2 z}{\partial x^2} + y \frac{\partial^2 z}{\partial x \partial y} = n \frac{\partial z}{\partial x}$

$\Rightarrow x \frac{\partial^2 z}{\partial x^2} + y \frac{\partial^2 z}{\partial x \partial y} = (n-1) \frac{\partial z}{\partial x}$ ----------- (2)

Differentiating (1) w.r.t y ,

$$x \frac{\partial^2 z}{\partial y \partial x} + 1. \frac{\partial z}{\partial y} + y \frac{\partial^2 z}{\partial y^2} = n \frac{\partial z}{\partial y}$$

$\Rightarrow x \frac{\partial^2 z}{\partial x \partial y} + y \frac{\partial^2 z}{\partial y^2} = (n-1)\frac{\partial z}{\partial y}$ ------------ (3)

{As $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ are continuous}

{$\Rightarrow \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$ }

Equ^n(2) X x + Equ^n(3) X y

$\Rightarrow x^2 \frac{\partial^2 z}{\partial x^2} + xy \frac{\partial^2 z}{\partial x \partial y} + xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = x(n-1) \frac{\partial z}{\partial x} + y (n-1)\frac{\partial z}{\partial y}$

$\Rightarrow x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = (n-1) \{ x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} \}$

= (n-1) nz

= n (n-1)z (proved)

Exercise

Question with short answers (2 marks)

1) if z = $\sin \frac{x}{y}$ find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

2) If f(x,y) = $\sqrt{x^2 + y^2}$, find f_x , f_y .

3) If f(x,y) = log ($x^2 + y^2$- 2xy) find f_{xx} , f_{yx} , f_{xy}

4) If z = f(x,y) , then find $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$

5) Find $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ if z = $xe^y + ye^x$

Questions with long answers (5 marks)

6) Given f(u,v) = $\frac{2u-3v}{u^2+v^2}$, find f_u (2,1) and f_v(2,1)

7) If z = $\frac{x-y}{x+y}$, then show that x $\frac{\partial z}{\partial x}$ + y $\frac{\partial z}{\partial y}$ = 0

8) If z = x^2y + 3xy^2 - $\frac{x}{y}$. Find partial derivatives of 2nd order.

9) Verify Euler's theorem for u = $x^2 \log(\frac{y}{x})$

10) If z = xy f ($\frac{y}{x}$) , then show that x $\frac{\partial z}{\partial x}$ + y $\frac{\partial z}{\partial y}$ = 2z

11) If u = $\sin^{-1}(\frac{x+y}{\sqrt{x}+\sqrt{y}})$ show that x $\frac{\partial u}{\partial x}$ + y $\frac{\partial u}{\partial y}$ = $\frac{1}{2}$ tan u

12) If z = $\ln(\frac{x^2+y^2}{x+y})$ then show that x $\frac{\partial z}{\partial x}$ + y $\frac{\partial z}{\partial y}$ = 1

13) If z = $\cos^{-1}(\frac{x^2+y^2}{x+y})$, then show that x $\frac{\partial z}{\partial x}$ + y $\frac{\partial z}{\partial y}$ = - cot z

Answers

1) $\frac{1}{y}$ cos $(\frac{x}{y})$, $\frac{-x}{y^2}$ cos $(\frac{x}{y})$ 2) $\frac{x}{\sqrt{x^2+y^2}}$, $\frac{y}{\sqrt{x^2+y^2}}$ 3) $\frac{-2}{(x-y)^2}$, $\frac{2}{(x-y)^2}$, $\frac{2}{(x-y)^2}$

4) y f'(xy) , xf'(xy) 5) e^y + y e^x , xe^y + e^x 6) $\frac{6}{25}$, $\frac{-17}{25}$

8) Z_{xx} =2y , Z_{yx} = 2x + 6y+ $\frac{1}{y^2}$, Z_{xy} = 2x + 6y+ $\frac{1}{y^2}$, Z_{yy} = 6x - $\frac{2x}{y^3}$.